(Pages : 4)

ALL SAINTS COLLEGE LIBRARY
No.....
TRIVANDRUM

K – 3230

Reg. No. : .....

Name : .....

Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1543 – DIFFERENTIAL EQUATIONS

(2015-2017 admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first** questions are compulsory. They carry **1** marks each.

1. Find the order of the differential equation $(y''')^2+(y'')^9+y=4$.
2. State whether the equation $y'+y^2 \tan x=\cos^3 x$ is linear or non-linear,
3. Verify that $y=c(1+x)$ is a solution of $(1+x)y'-y=0$.
4. Give the general form of a separable equation.
5. Define growth constant.
6. A differential equation $M\,dx+N\,dy$ is exact if and only if ————————
7. Write down the standard form of a linear differential equation of second order.
8. Give an example for an Euler-Cauchy equation.

P.T.O.

9. Write down the auxiliary equation of the differential equation $y''+y'-3y=0$.

10. State the Existence and Uniqueness Theorem for initial value problem.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the question 11 to 22. These questions carry **2** marks each.

11. Solve $x(1+y^2)\,dx+y(1+x^2)\,dy=0$.

12. Solve $y'+y^2\sin x=0$.

13. Solve $y'+3y=e^{-2x}$.

14. Find a curve in the xy-plane that passes through $(0,3)$ and whose tangent line at a point (x,y) has slope $\frac{2x}{y^2}$.

15. Solve the differential equation $(x^3+3xy^2)\,dx+(y^3+3x^2y)\,dy=0$.

16. Find a general solution of $y''-y'-6y=0$.

17. Find a general solution of $x^2y''-\frac{5}{2}xy'-2y=0$.

18. Find the Wronskian of the bases $\{\cos ax, \sin ax\}$.

19. Write down the auxiliary equation of the differential equation $x^2y''+6xy'+4y=0$.

20. Verify $y_p=e^{-3x}$ is a solution of $y''-y=8e^{-3x}$.

21. Find a general solution of $x^2y''+7xy'+13y=0$.

22. Define general solution of nonhomogeneous linear differential equation.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** question from among the questions 23 to 31. These questions carry **4** marks each.

23. Solve the initial value problem $xy'+y=x;\ y(1)=2.$

24. Solve the initial value problem $y'=\frac{4x^2}{y+\cos y};\ y(1)=\pi.$

25. According to United Nations data, the world population in 1998 was approximately 5.9 billion and growing at a rate of about 1.33% per year. Assuming an exponential growth model, estimate the world population at the beginning of the year 2023.

26. Find an integrating factor and solve the differential equation $(2\cos y++4x^2)dx=x\sin y\,dy.$

27. Using the method of reduction of order solve the differential equation $x^2y''-5xy'+9y=0$, given that $y_1=x^3$ is a solution.

28. Solve the initial value problem $y''+4y'+4y=0;\ y(0)=1;\ y'(0)=1.$

29. Solve the initial value problem $x^2y''+xy'+9y=0;\ y(1)=2;\ y'(1)=0.$

30. Solve the differential equation $y''-4y'+5y=e^{2x}\csc x.$

31. Solve the initial value problem $4x^2y''+24xy'+25y=0;\ y(1)=2;\ y'(1)=-6.$

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. (a) Solve the initial value problem $(y+2)dx+y(x+4)dy=0; y(-3)=-1$.

    (b) Use Euler's Method with a step size of 0.1 to make a table of approximate values of the solution of the initial value problem $y'=y-x; y(0)=2$, over the interval $0 \le x \le 1$

33. At time t = 0, a tank contains 4 lb of salt dissolved in 100 *gal* of water. Suppose that brine containing 2 *lb* of salt per gallon of brine is allowed to enter the tank at a rate of 5 *gal/min* and that the mixed solution is drained from the tank at the same rate. Find the amount of salt in the tank after 10 minutes.

34. (a) Solve the differential equation $y''+y=\sec x$ using the Method of Variation of Parameters.

    (b) Solve the boundary value problem $y''+y=0; y(0)=3; y(\pi)=-3$.

35. Solve the following differential equations using the Method of Undetermined Coefficients.

    (a) $y''+4y=8x^2$

    (b) $y''+2y'+y=e^{-x}$.

(2 × 15 = 30 Marks)
